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1999 J. Phys.: Condens. Matter 11 513

(http://iopscience.iop.org/0953-8984/11/2/015)

J. Phys.: Condens. Matter **11** (1999) 513–520. Printed in the UK PII: S0953-8984(99)95438-3

Internal oscillations of kink-type solitons in one-dimensional antiferromagnets like TMMC

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Received 29 June 1998, in final form 19 October 1998

Abstract. The problem of internal oscillations of kink-type solitons in a one-dimensional easy-plane antiferromagnet is studied by analytical methods. Apart from the Goldstone mode a second local mode which is due to the coupling between in-plane and out-of-plane spin components has been derived. For typical experimental conditions in a model system like $(CH_3)_4NMnCl_3$ (TMMC) the frequency of this mode is very close to the bottom of the magnon band. Considering both a twofold and a weak sixfold in-plane crystal field anisotropy we found that close to the spin-flop transition the separation of this local mode from the magnon band is increased and dramatically affected by the sixfold anisotropy. We show that quantum properties of this mode are not important.

1. Introduction

The important role of solitons in the physics of one-dimensional (1D) magnets is well known. The specific contribution of 1D-solitons (kinks) to thermodynamic characteristics, especially to response functions, is clearly established in a variety of experiments, see reviews [1–5].

The most direct way of detecting solitons is to observe their contributions to the cross section of inelastic neutron scattering or to the relaxation rate of nuclear magnetic resonance [1, 2]. These contributions mainly result from the flipping of spins connected with the motion of the solitons. Formally this motion can be described in terms of a quasi-elastic (translational) mode and manifests itself as a central peak at zero frequency. In addition to the translation mode solitons can have other internal degrees of freedom, which are described by magnon modes localized to the soliton (*local modes*) [5]. These modes should be of nonzero frequency and, therefore, observable by standard spin resonance technique. For the highly anisotropic 1D quantum antiferromagnet $CsCoCl_3$ the internal mode was observed by inelastic neutron scattering and electron paramagnetic resonance [6]. Note that this Ising-type magnet as well as Haldane systems [1] are essentially quantum systems whose properties differ strongly from 'classical' soliton-bearing magnets like $CsNiF_3$ or TMMC, which are usually associated with the soliton physics of 1D magnets.

It is well known that in easy-plane antiferromagnets two types of magnon occur, *in-plane* magnons oscillating mainly parallel to the easy plane and *out-of-plane* magnons oscillating perpendicularly to it. A similar distinction holds for local modes, too. The out-of-plane *internal* mode corresponds to spin oscillations out of the easy plane which are localized to the kinks. This mode was considered many years ago [7] and observed for a system of domain walls in thin plates of thulium orthoferrite [8]. Recently it was shown [9] that this out-of-plane mode for 1D antiferromagnets has essentially quantum nature, even for spin $S = 5/2$, which

0953-8984/99/020513+08$19.50

is usually considered as semiclassical. For easy-plane antiferromagnets like TMMC out-of-plane magnons are generally of much higher frequency than in-plane magnons, which interact with the solitons and are manifested in soliton–magnon interference [10]. Such oscillations are usually described in terms of a sine–Gordon (SG) equation. This equation, however, shows no local mode of nonzero frequency. Therefore, the occurrence of such modes in easy-plane magnets requires a treatment beyond the SG limit [11].

This article is aimed at the analysis of in-plane local modes for a real model system like TMMC beyond the usual sine–Gordon approximation. We define the parameter range where the in-plane local mode occurs. In contrast to out-of-plane modes, quantum effects are negligible here.

2. Model and elementary excitations

Quasi-one-dimensional Heisenberg antiferromagnets (AFMs) like TMMC are described by the spin Hamiltonian

$$\mathcal{H} = \sum_i \{J\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + K(S_i^z)^2 + K_2(S_i^y)^2 - g\mu_B \boldsymbol{H}\cdot\boldsymbol{S}_i\} + \mathcal{H}_6. \tag{1}$$

The $\boldsymbol{S}_i$ are the atomic spins located on a 1D lattice with lattice constant a, $J > 0$ is the exchange integral, and K and K_2 are anisotropy constants describing the strong easy-plane anisotropy and a weak in-plane anisotropy resulting from rhombic distorsions at lower temperature. $\boldsymbol{H}$ is the magnetic field, μ_B the Bohr magneton and g the Landé factor of the magnetic ion. For TMMC we have $J/k_B = 13.6$ K, $K/k_B \simeq 0.3$ K and $K_2/k_B \simeq 4$ mK at $T = 4.2$ K [2, 12]. $\mathcal{H}_6$ finally denotes a very weak hexagonal anisotropy, which has not been considered in previous calculations and will be discussed and evaluated below.

It is convenient to describe macroscopic excitations like long-wave magnons or kink-type solitons in terms of the nonlinear σ-model (see reviews [1, 4]). This model is written for a unit vector field $\boldsymbol{l} = \boldsymbol{l}(z, t)$, which denotes the normalized sublattice magnetization in the continuum limit: $\boldsymbol{l} \equiv (\boldsymbol{S}_{i+1} - \boldsymbol{S}_i)/2S$. The net magnetization of the AFM, $\boldsymbol{m} \equiv (\boldsymbol{S}_i + \boldsymbol{S}_{i+1})/2S$, can be written as a function of $\boldsymbol{l}$ and $\partial\boldsymbol{l}/\partial t$. Changing over to spherical coordinates $l_z = \cos\theta$ and $l_x + \mathrm{i}l_y = \sin\theta\exp(\mathrm{i}\varphi)$ two coupled equations for θ and φ are obtained.

This set of equations was used to analyse the kink dynamics in presence of a strong magnetic field for rhombic and uniaxial AFMs [13–15]. An exact analytical solution is still missing, but numerical solutions [13] and analytical approximations [14, 15] could be obtained. For the experimentally realistic case of small in-plane anisotropy $K_2 \ll K$ and for a magnetic field far below the out-of-plane instability $H \ll H_c \equiv S\sqrt{8JK}/g\mu_B$, the oscillation of $\boldsymbol{l}$ has almost in-plane character, i.e. $\theta \simeq \pi/2$. Then the equations of motion for θ and φ can essentially be simplified. Introducing the small parameters K_2/K and H/H_c the resulting equation of motion can be written in a linear approximation as a function of $\varphi = \varphi(z, t)$ only and takes the form of a generalized sine–Gordon (GSG) equation:

$$\frac{1}{c^2}\ddot{\varphi} - \varphi'' + \frac{1}{Ja^2S^2}\frac{\partial w_a}{\partial\varphi} + \frac{4}{c^2}\frac{H^2}{H_c^2}[\dot{\varphi}\cos(\varphi-\gamma)]\cdot\cos(\varphi-\gamma) = 0. \tag{2}$$

Here $c = 2JSa\hbar^{-1}$ is the magnon velocity in the isotropic limit ($w_a = 0$ and $\boldsymbol{H} = \boldsymbol{0}$), and the prime and dot denote space and time derivatives. $w_a(\varphi)$ describes the effective anisotropy energy within the easy plane, including $\mathcal{H}_6$,

$$w_a(\varphi) = S^2K_2\sin^2\varphi + S^2K_6\sin^2 3\varphi + [(g\mu_BH)^2/8J]\cos^2(\varphi-\gamma). \tag{3}$$

The sixfold anisotropy reflecting the hexagonal symmetry of the crystal lattice has not been considered so far, but its account is crucial for the local mode. The last term in (3) describes the effective anisotropy imposed by a magnetic field which is applied in the easy plane. γ is the angle between $\boldsymbol{H}$ and the easy axis (denoted by $\varphi = 0, \pi$, see figure 1). Note that the role of the magnetic field is twofold: First, to renormalize the anisotropy energy, equation (3), which gives rise to the well-known spin-flop transition. Second, to break the Lorentz invariance present at $H = 0$ (the last term in equation (2)) which is important for local mode features. The characteristic value H_c related to out-of-plane instability is much larger than the spin-flop field H_{SF}

$$g\mu_B H_c \equiv S\sqrt{8JK} \gg S\sqrt{8JK_2} \equiv g\mu_B H_{SF} \tag{4}$$

and describes the gap of the highest (out-of-plane) magnon branch. For TMMC we have $H_c \simeq 100$ kOe and $H_{SF} \simeq 12$ kOe [10].

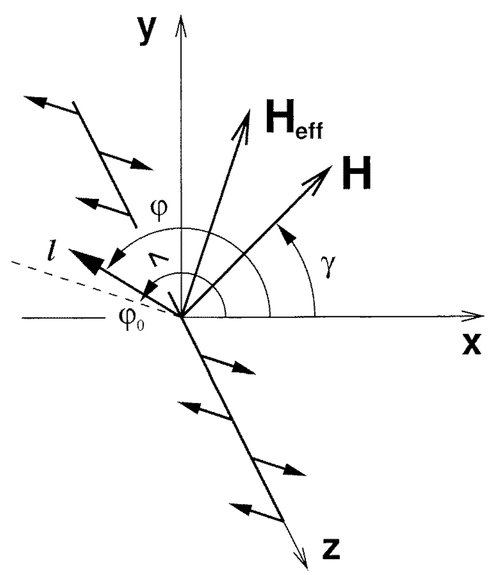

Figure 1. Orientation of the antiferromagnetic spin chain ($\boldsymbol{z}$), easy K_2 axis ($\boldsymbol{x}$), resulting effective field $\boldsymbol{H}_{eff}$, and local sublattice magnetization $\boldsymbol{l}$. The ground-state orientation φ_0 of the spins is perpendicular to $\boldsymbol{H}_{eff}$.

The usual elementary excitations important for 1D magnets, magnons and kinks, can easily be extended to the GSG case, equation (2). We will briefly discuss only those properties important for the description of localized modes. The magnon gap frequency $\tilde{\omega}_g$ is renormalized with respect to the well known SG result $\omega_g = g\mu_B H_{eff}\hbar^{-1}$ and takes the form

$$\hbar^2\tilde{\omega}_g^2 = [(g\mu_B H_{eff})^2\cos^2(\varphi_0 - \alpha) + 72JK_6S^2\cos 6\varphi_0]/[1 + 4(H/H_c)^2\cos^2(\varphi_0 - \gamma)]. \tag{5}$$

Here we used the notation $H_{eff}^4 \equiv H^4 - 2H^2\,H_{SF}^2\cos 2\gamma + H_{SF}^4$, which determines the magnon gap in the SG limit $K_2/K \to 0$ and $K_6 = 0$ [10]. φ_0 denotes the ground state orientation of $\boldsymbol{l}$ for arbitrary values of K_2 and K_6 (see figure 1). In the linear approximation for K_6/K_2 it can be expressed by the simple relation

$$\varphi_0 = \alpha - 1.5(K_6/K_2)(H_{SF}/H_{eff})^2\sin 6\alpha \tag{6}$$

where α finally denotes the ground state orientation in the limit $K_6 = 0$ and is determined by the relations (see also [11])

$$\sin 2\alpha = -H^2\sin 2\gamma/H_{eff}^2 \qquad \cos 2\alpha = (H_{SF}^2 - H^2\cos 2\gamma)/H_{eff}^2. \tag{7}$$

For large magnetic fields $H_{SF} \ll H \ll H_c$, including equations (6) and (7), the magnon gap energy takes the simple form

$$\hbar^2\tilde{\omega}_g^2 \simeq g^2\mu_B^2[H^2 - H_{SF}^2 \cos 2\gamma - 9H_{SF}^2(K_6/K_2)\cos 6\gamma]. \tag{8}$$

This dependence could be used for the evaluation of K_6 from experimental data. The angular dependence of the resonance frequency $\tilde{\omega}_g$ was measured in an AFMR experiment on TMMC at 1.7 K [12]. Apart from the superposition of three independent and well pronounced periodic functions corresponding to the three different orientations of crystalline domains [10] we found a slight deviation from the expected harmonic dependence, which could be attributed to the third term in equation (8). By comparison with the theoretical expression we obtained the estimate $K_6/k_B \simeq 0.04$ mK.

The kink solutions of equation (2) mainly resemble the usual SG results, but again with renormalized parameters. The shape of an unmoving kink Φ_0 can be described by $\varphi = \varphi_0 + \Phi_0(z)$, where $\Phi_0 \to 0, \pi$ for $z \to \mp\infty$. In a first approximation for K_6 the function $\Phi_0(z)$ is obtained by integration of the expression

$$\left(\frac{\mathrm{d}\Phi_0}{\mathrm{d}x}\right)^2 = \sin^2\Phi_0[1 + \epsilon_1 \cos 6\alpha(3 - 4\sin^2\Phi_0)^2 - 16\epsilon_1 \sin 6\alpha \sin\Phi_0 \cos^3\Phi_0] \tag{9}$$

where the space coordinate has been scaled by the width of the kink: $x = z/\Delta_0$ with $\Delta_0 \equiv a(J/2K_2)^{1/2}H_{SF}/H_{eff}$, and the small parameter $\epsilon_1 \equiv (K_6/K_2)(H_{SF}/H_{eff})^2$ has been introduced. For vanishing hexagonal anisotropy $\epsilon_1 = 0$ the usual SG result $\cos\Phi_0 = \tanh(x - x_0)$ is retained.

3. Magnon modes localized on the kink

The occurrence of a local mode of nonzero frequency is a highly nontrivial property of kinks in different models. For example, such a mode is absent in the SG model, but a well known property of the double-sine–Gordon or the φ^4 model [4, 5]. For 1D AFMs a strong magnetic field parallel to the easy plane can also give rise to such a local mode [11]. We will show that the kinks of the GSG equation (2) have a non-translational local mode which for $\epsilon_1 \ll 1$ and $H \ll H_c$ occurs inside the magnon gap near the bottom of the continuum. To analyse this local mode we start from the decomposition $\varphi(z,t) = \varphi_0 + \Phi_0(z) + \psi(z,t)$, where φ_0 denotes the ground state, $\Phi_0(z)$ describes a kink at rest, and $\psi(z,t)$ a small oscillating deviation from this kink, which allows us to linearize equation (2) with respect to $\psi(z,t)$. Assuming a harmonic oscillation $\sim \cos\omega t$, we obtain the eigenvalue problem

$$\mathcal{L}\psi = \frac{\omega^2}{\omega_0^2}[1 + 4(H/H_c)^2\cos^2(\Phi_0 + \varphi_0 - \gamma)]\psi \tag{10}$$

where $\hbar\omega_0 \equiv g\mu_B H_{eff}$ is the magnon gap in the limit H/H_c, $\epsilon_1 \to 0$. The Schrödinger operator $\mathcal{L}$ on the l.h.s. has the meaning

$$\mathcal{L} \equiv -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \{\cos 2\Phi_0 + 9\epsilon_1 \cos 6(\Phi_0 + \varphi_0)\}. \tag{11}$$

The space-dependent (via $\Phi_0(z)$) term on the r.h.s. of equation (10) is proportional to the small parameter $\epsilon_2 \equiv (H/H_c)^2$.

For vanishing hexagonal anisotropy $\epsilon_1 = 0$ the eigenvalue problem (10) can approximately be reduced to the usual Schrödinger eigenvalue problem with a Pöschl–Teller potential $V(x) = -a/\cosh^2 x$ with well known properties [11]. For $\epsilon_1 \neq 0$ this method no longer holds, and—apart from obtaining the trivial solution for the translational Goldstone mode by $\psi_0(x) = \mathrm{d}\Phi_0(x)/\mathrm{d}x$—a different analysis has to be applied.

For analysing the next nontrivial local mode we use a method developed in [16, 17], which can be applied to all magnetic systems differing weakly from the SG model. The existence of the translational mode $\psi_0(x)$ allows us to present the Schrödinger operator in the factorized form $\mathcal{L} = \mathcal{L}^{(+)}\mathcal{L}^{(-)}$, where

$$\mathcal{L}^{(+)} \equiv \frac{\mathrm{d}}{\mathrm{d}x} + \frac{\psi_0'}{\psi_0} \qquad \mathcal{L}^{(-)} \equiv -\frac{\mathrm{d}}{\mathrm{d}x} + \frac{\psi_0'}{\psi_0}. \tag{12}$$

Note that $\mathcal{L}^{(-)}\psi_0 = 0$. Introducing the new variable $\tilde{\psi} \equiv \mathcal{L}^{(-)}\psi$ the eigenvalue problem (10) takes the form

$$\mathcal{L}^{(-)}[1 + \epsilon_2\, 4\cos^2(\varphi - \gamma)]\mathcal{L}^{(+)}\tilde{\psi} = (\omega/\omega_0)^2\tilde{\psi}. \tag{13}$$

On first glance, this equation does not look better than (10). But for our case (and for all such cases where the equation of motion does not strongly differ from the SG model) it turns out to be very convenient. Note that for $\epsilon_1 = 0$ the operator $\mathcal{L}^{(-)}\mathcal{L}^{(+)}$ is reduced to the potentialless form $[-\mathrm{d}^2/\mathrm{d}x^2 + 1]$. So, for vanishing ϵ_1 and ϵ_2 the eigenvalue problem becomes trivial. For small nonzero values of ϵ_1 and ϵ_2 the new eigenvalue problem (13) has no Goldstone mode, and the local mode (if it exists) must be the lowest one. We then expect that the frequency of this local mode ω_l should be close to the magnon gap $\tilde{\omega}_g$, i.e. $\tilde{\omega}_g - \omega_l \ll \tilde{\omega}_g$. By introducing the new variable $\xi \equiv [1 + \epsilon_2\, 4\cos^2(\varphi - \gamma)]^{1/2}\tilde{\psi}$ and using the concrete form of $\psi_0(x)$, we arrive after some lengthy algebra at a Schrödinger-like equation for ξ. In first-order approximation for ϵ_1 and ϵ_2 it can be written in the form

$$-\frac{\mathrm{d}^2\xi}{\mathrm{d}x^2} + \{1 + \epsilon_1 V_1(x) + \epsilon_2 V_2(x)\}\xi = \frac{\omega^2}{\tilde{\omega}_g^2}\xi. \tag{14}$$

The 'potentials' V_1 and V_2 have both symmetrical and antisymmetrical parts, for example

$$V_2(x) = \tfrac{1}{4}\sin^2 2\Phi_0 \cos 2(\gamma - \alpha) - \tfrac{1}{8}\sin 4\Phi_0 \sin 2(\gamma - \alpha). \tag{15}$$

For $V_1(x)$ we present only the symmetrical part important for the local mode analysis,

$$\tfrac{1}{2}[V_1(x) + V_1(-x)] = -128\sin^4\Phi_0 \cos^2\Phi_0 \cos 6\alpha. \tag{16}$$

In this approximation the exact ground state orientation φ_0 can be replaced by its approximate value α, and the soliton shape function $\Phi_0(x)$ can be expressed by the SG result $\sin\Phi_0 = 1/\cosh x$. Thus, we have reduced the eigenvalue problem of equation (10) to that of an ordinary Schrödinger equation with the small potential $V = \epsilon_1 V_1 + \epsilon_2 V_2$. From general properties of such operators it follows that they can have only one local mode. For this possible local mode the eigenfrequency ω_l is close to the gap frequency $\tilde{\omega}_g$. It is also known that the eigenfunction of this mode is localized in the region $\Delta x = [\tilde{\omega}_g/(\tilde{\omega}_g - \omega_l)]$. For the present situation this value is much larger than the kink width Δ_0 or the localization of potentials $V_1(x)$ and $V_2(x)$. This means that for the analysis of this mode we can replace them by δ-functions, $\epsilon_1 V_1(x) + \epsilon_2 V_2(x) - V_0\Delta_0\delta(x)$, where the coefficient V_0 is given by the integral

$$V_0 = -\int_{-\infty}^{\infty} \mathrm{d}x\{\epsilon_1 V_1(x) + \epsilon_2 V_2(x)\} = \frac{512}{15}\epsilon_1 \cos 6\alpha - \frac{8}{3}\epsilon_2 \cos 2(\gamma - \alpha). \tag{17}$$

The occurrence and properties of the local mode are determined by the sign and amplitude of the substitute δ-potential. (This is the reason why antisymmetrical parts of $V_1(x)$ and $V_2(x)$ are unimportant. They do not contribute to V_0, but only affect higher order contributions.) The local mode appears only for $V_0 > 0$, which corresponds to the condition that the effective potential has to be attractive. The approximation by a δ-function leads to solutions of the form

$$\xi = \mathbb{C}\exp\left\{-|x|\sqrt{(\tilde{\omega}_g^2 - \omega_l^2)/\tilde{\omega}_g^2}\right\}. \tag{18}$$

From the condition at $x = 0$, $(\mathrm{d}\xi/\mathrm{d}x)|_{x\to+0} - (\mathrm{d}\xi/\mathrm{d}x)|_{x\to-0} = V_0$ we obtain the final expression for the frequency of the local mode,

$$\omega_l = \tilde{\omega}_g\sqrt{1 - \tfrac{1}{4}V_0^2}. \tag{19}$$

Transforming solution (18) back to the variable $\tilde{\psi}(x)$ and solving the simple equation $\tilde{\psi} = \mathcal{L}^{(-)}\psi$ the corresponding eigenfunction is obtained. In the lowest approximation on small parameters we have

$$\psi(z,t) = q_0\cos(\omega t + \delta_0)\tanh(z/\Delta_0)\exp(-\sqrt{2(\tilde{\omega}_g - \omega_l)/\tilde{\omega}_g}|z/\Delta_0|) \tag{20}$$

where q_0 and δ_0 have the meaning of arbitrary amplitude and phase of the oscillation. The normalized frequency difference $(\tilde{\omega}_g - \omega_l)/\tilde{\omega}_g$ is proportional to squared small parameters. It can be presented in the convenient form

$$\sqrt{2(\tilde{\omega}_g - \omega_l)/\tilde{\omega}_g} = \frac{V_0}{2} = \frac{256}{15}\epsilon_1\cos 6\alpha - \frac{4}{3}\epsilon_2\cos 2(\gamma - \alpha) \tag{21}$$

which reflects the fact that both the difference $\tilde{\omega}_g - \omega_l$ and the amplitude V_0 (the r.h.s. of equation (21)) have to be positive for the occurrence of the local mode.

The two competing terms differ strongly in their field and angular dependencies. (For detailed discussion it is convenient to use the normalized field $h \equiv H/H_{SF}$.) Considering the low field limit $H^2 \ll H_{SF}^2$ (i.e. $h \ll 1$), we have $\alpha \simeq 0$ and $\epsilon_1 \simeq K_6/K_2 \simeq 10^{-2}$. For the case of TMMC we thus obtain the value $(0.17 - 0.022h^2\cos 2\gamma)$, which is positive for $h \leqslant 1$, and the local mode resulting from the hexagonal anisotropy is present.

In the high field limit $H^2 \gg H_{SF}^2$ $(h^2 \gg 1)$, the sublattice magnetization is nearly perpendicular to the magnetic field $(\alpha - \gamma \simeq \pi/2)$ and both terms are positive, but now only the second one, which has already been calculated in [11], will be important. Accordingly, the frequency difference $\tilde{\omega}_g - \omega_l$ increases proportional to h^4 and is independent of field orientation γ.

The local mode should show up most distinctly close to the spin-flop transition, where the competition between the hexagonal and the field-induced anisotropies is most pronounced. Since the parameter ϵ_1 is determined by the ratio of the hexagonal anisotropy K_6 and the squared effective magnon gap which is proportional to H_{eff}^2 and becomes very small for $H \to H_{SF}$ the effect of K_6 will be strongly enhanced. Close to H_{SF} the parameter ϵ_1 can be presented in the form

$$\epsilon_1 = \frac{K_6}{K_2}[(1 - h^2)^2 + 4h^2\sin^2 2\gamma]^{-1/4}. \tag{22}$$

For $\gamma \simeq 0, \pi$ and $h \to 1$ the value of ϵ_1 is strongly increased. Note, however, that the theory still holds for $\epsilon_1 \ll 1$ only. For TMMC this condition is guaranteed for $1 - h^2 \gg 10^{-2}$. In this case even the sign of K_6 is not important for the occurrence of the local mode, since the sublattice orientation α changes at H_{SF} from 0 to $\pi/2$. In both cases the sign of the decisive term $K_6\cos 6\alpha$ can be positive approaching H_{SF} either from below (for $K_6 > 0$) or from above (for $K_6 < 0$). Thus, we see that the magnon mode localized on the kink can appear for small enough but nonzero values of the hexagonal anisotropy.

Finally, we discuss possible quantum properties of this mode. It was recently shown [4, 8] that the local mode with l-vector oscillations out of the easy plane and frequencies far above the gap of the in-plane magnons $\tilde{\omega}_g$ should be considered as a quantum state. E.g. its zero-point fluctuations are not small; their amplitude is of order one. Therefore, we should also consider the effect of zero-point fluctuations for the in-plane local mode discussed above.

Following the simple procedure used in [4], we take the variable $\varphi(z,t)$ corresponding to the local mode of the form (20), replacing q_0 by $q(t)$, and insert it into the Lagrangian of the

GSG equation. In the linear approximation the effective Lagrangian for $q(t)$ takes the standard form of a harmonic oscillator $\mathcal{L}_{eff} = I(\dot{q}^2 - \omega_l^2 q^2)/2$. The 'effective mass' (the momentum of inertia) reads $I = \hbar S[8(\tilde{\omega}_g - \omega_l)\tilde{\omega}_g]^{-1/2}$. The m.s.r. amplitude of zero-point fluctuations can be obtained from the standard relation $\langle q^2 \rangle = \pi\hbar/2I\tilde{\omega}_g$. Combining these two relations we see that this value is independent of specific system parameters, as was the case for the out-of-plane mode:

$$\langle q^2 \rangle = \frac{\pi}{S}\sqrt{2(\tilde{\omega}_g - \omega_l)/\tilde{\omega}_g}. \tag{23}$$

It only depends on the normalized frequency shift of the local mode with respect to the magnon gap. That means, for the weakly localized mode considered above $\langle q^2 \rangle$ is small and quantum effects are unimportant for the description of its dynamics.

4. Conclusions

We have discussed the properties of internal oscillations of kink-type solitons occurring in quasi-one-dimensional easy-plane antiferromagnets like TMMC above T_N. These oscillations can be interpreted as a non-Goldstone soliton–magnon bound state arising from the coupling between in-plane and out-of-plane spin components. The local mode emerges from the bottom of the spin-wave band. Neglecting the hexagonal anisotropy ($K_6 = 0$) its frequency separation from the lowest (i.e. uniform) magnon was found to increase asymptotically like H^4, but still remains rather small for TMMC and typical experimental conditions. Including only a weak hexagonal anisotropy ($K_6 \neq 0$) results in a dramatic enhancement of this separation, especially when H is close to the spin-flop field. Finally, in contrast to previous results on out-of-plane modes, we have shown that quantum properties of this in-plane local mode are not important.

Since the local mode carries some nonzero net magnetization, it can be probed by standard electron spin resonance (ESR) experiments. It was earlier shown [11] that the polarization of this magnetization depends on the direction and magnitude of the magnetic field $\boldsymbol{H}$ and differs significantly from that of the usual uniform ESR mode. Both the different resonance frequency and different polarization of this mode represent signatures which can be probed in experiment. According to our calculation, the most promising condition for obtaining experimental evidence is to measure its resonance frequency close to the spin-flop field. We hope to stimulate future experiments which confirm (or refute) these theoretical findings.

Acknowledgments

This project of SFB 185 'Nichtlineare Dynamik' was partly financed by special funds of the Deutsche Forschungsgemeinschaft. We thank J Hołyst for useful comments.

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